

# GAUGINO CONDENSATION AND SUSY BREAKDOWN

Hans Peter Nilles

*Physikalisches Institut, Universität Bonn,  
Nussallee 12, D-53115 Bonn, Germany*

nilles@th.physik.uni-bonn.de

## Abstract

We review the mechanism of gaugino condensation in the framework of the  $d = 10$  heterotic string and its  $d = 11$  extension of Horava and Witten. In particular we emphasize the relation between the gaugino condensate and the flux of the antisymmetric tensor fields of higher dimensional supergravity. Its potential role for supersymmetry breakdown and moduli stabilization is investigated.

## Keywords:

Supersymmetry, Supergravity, String theory, Gaugino condensation, Spontaneous breakdown of supersymmetry

## 1. Introduction

The topic of my lectures at the Cargèse summer school 2003 was a general introduction to the breakdown of supersymmetry in field- and string-theory. In this written up version I decided to concentrate on some particular aspects of this mechanism: gaugino condensation in the framework of heterotic string- and M-theory. This allows a more detailed discussion of supergravity in  $d = 10$  and  $d = 11$  dimensions and the reduction to the  $d = 4$  case.

The mechanism of gaugino condensation is believed to play a crucial role for moduli stabilization and SUSY breakdown in string theory. Conceived as a mechanism for hidden sector supersymmetry breakdown in supergravity extensions of the standard model of strong and electroweak interactions [92, 93, 40] it found a natural setting in the framework of

the  $E_8 \times E_8$  heterotic string theory [46] (see ref. [28, 32]) as well as the M-theory of Horava and Witten [53] (suggested in [52, 101]).

One of the attractive features of the mechanism is a specific cancellation of  $H$ -flux and the gaugino condensate in the low energy effective potential. This allows a somewhat controlled discussion of the vacuum energy at least at the classical level and it is the direct consequence of the properties of the higher dimensional supergravity action. It also emphasizes the importance of Chern-Simons-terms in  $H$ -flux of  $d = 10$  supergravity [29].

In these lectures we shall not attempt to construct a fully realistic model but try to explain the mechanism in its simplest form. In section 2 we will discuss supergravity in  $d = 10$  and then define the supermultiplets relevant for the  $d = 4$  discussion. The mechanism of gaugino condensation is introduced in section 4 followed by a determination of the  $d = 4$  effective action using the method of reduction and truncation [114]. Section 6 contains some aspects of the theory beyond the classical level. In section 7 we give a detailed discussion of the  $d = 11$  heterotic M-theory and the determination of the low energy effective actions. Some aspects of the mechanism that were rather obscure in the  $d = 10$  theory (like the cancellation of gaugino bilinears due to Chern-Simons flux) become obvious in this generalized picture [102]. Section 8 will then focus on the specific properties of supersymmetry breakdown at the hidden wall. The last section discusses some recent developments that lead to a revived interest of this mechanism during the last year.

Before we start let me make a technical comment. Traditionally there are two different ways to include a gaugino condensate in the effective action. The first one [40, 28] uses explicitly the  $F$ -terms of the supersymmetry transformation laws. These include gaugino bilinears multiplied by the derivative of the gauge kinetic function. These gaugino bilinears are then replaced by the (field dependent) renormalization group invariant scale and included in the standard fashion in the scalar potential. This is the procedure which we follow in these lectures. An alternative method [32, 29] postulates the gaugino bilinear as a new term in an effective superpotential. The qualitative results of the two mechanisms are the same. Some quantitative differences will be mentioned where it applies.

## 2. Supergravity in $d = 10$

We shall here discuss the effective action of superstring theories in the supergravity field theory framework. For the known ( $N = 1$ ) superstring theories this is  $N = 1$  supergravity in  $d = 10$  coupled to pure  $E_8 \times E_8$

or  $0(32)$  gauge multiplets. The spectrum of this theory is given by the supergravity multiplet  $(g_{MN}, \psi_{M\alpha}, B_{MN}, \lambda_\alpha, \phi)$  where  $M, N = 0, \dots, 9$  are world indices and  $\alpha$  is a Majorana-Weyl spinor index, as well as the gauge multiplet  $(A_M^A, \chi_\alpha^A)$  where  $A = 1, \dots, 496$  labels the adjoint representation of  $E_8 \times E_8$  or  $0(32)$ . In the Type I theory, these correspond to the massless closed (open) string states respectively. The action of such a theory, including terms up to two derivatives, is unique and given by [16]:

$$\begin{aligned}
e_{10}^{-1} \mathcal{L} = & -\frac{1}{2}R - \frac{i}{2}\bar{\psi}^M \Gamma^{MNP} D_N(\omega) \psi_P + \frac{9}{16} \left( \frac{\partial_M \varphi}{\varphi} \right)^2 + \\
& + \frac{3}{4} \varphi^{-3/2} H_{MNP} H^{MNP} + \frac{i}{2} \bar{\lambda} \Gamma^M D_M \lambda + \frac{3\sqrt{2}}{8} \bar{\psi}_M \left( \frac{\Gamma^P \partial_P \varphi}{\varphi} \right) \Gamma^M \lambda \\
& - \frac{\sqrt{2}}{16} \varphi^{-3/4} H_{MNP} \left( i \bar{\psi}_Q \Gamma^{QMNP R} \psi_R + 6i \bar{\psi}^M \Gamma^N \psi^P + \right. \\
& \left. + \sqrt{2} \bar{\psi}_Q \Gamma^{MNP} \Gamma^Q \lambda - i \bar{\chi} \Gamma^{MNP} \chi \right) - \\
& - \frac{1}{4} \varphi^{-3/4} F_{MN} F^{MN} + \frac{i}{2} \bar{\chi} \Gamma^M D_M(\omega) \chi - \\
& - \frac{i}{4} \varphi^{-3/8} \left( \bar{\chi} \Gamma^M \Gamma^{NP} F_{NP} \right) \left( \psi_M + \frac{i\sqrt{2}}{12} \Gamma_M \lambda \right) \\
& + \text{four fermion interactions}
\end{aligned} \tag{1}$$

where  $\Gamma$  denote Dirac matrices in  $d = 10$  and

$$F_{MN}^A = \frac{1}{2} \partial_{[M} A_{N]}^A + f^{ABC} A_M^B A_N^C \tag{2}$$

(written for short as  $F = dA + A^2$ ) denotes the gauge field strength. Supersymmetry requires the field strength  $H_{MNP}$  of the antisymmetric tensor field  $B_{MN}$  not just to be the curl of  $B$ , but

$$H_{MNP}^A = \partial_{[M} B_{NP]}^A + \omega_{MNP}^{YM} \tag{3}$$

where the Chern-Simons term is given by

$$\omega^{YM} = Tr \left( AF - \frac{2}{3} A^3 \right) \tag{4}$$

i.e.,  $B_{NP}$  has to transform non-trivially under the  $E_8 \times E_8$  [or  $0(32)$ ] gauge transformations. This theory as it stands has gravitational anomalies and is too naive an approximation to the anomaly-free superstring theory. The absence of anomalies requires an additional term to (3)[49]:

$$H = dB + \omega^{YM} - \omega^L \tag{5}$$

with

$$\omega^L = Tr(\omega R - \frac{2}{3}w^3) \quad (6)$$

where  $\omega_M^{ab}$  is the spin connection.  $\omega$  contains a derivative, thus  $\omega^L$  contains three and appears squared in the action. This term is purely bosonic and for a supersymmetric action requires additional terms which up now are only partially known. The action in (1) thus requires further terms in order to be an adequate low-energy limit of string theory. The action (1) was derived by truncating all heavy string states. For a better approximation they should be integrated out, leaving a low-energy theory with higher derivatives and terms in a higher order in  $\alpha'$  (the slope parameter). These terms appear in what is usually called " $\sigma$ -model perturbation theory", not to be confused with the string loop expansion, which, at least in the heterotic case, is an expansion in  $g$ , the gauge coupling constant. This expansion in powers of  $\alpha'$  is classical at the string level. There might also be world-sheet non-perturbative effects that play a role at this classical level. Looking at (1), one might wonder what  $g$  (the gauge coupling constant) is. Observe that the gauge fields have non-minimal gauge kinetic terms. Here  $g$  is not an input parameter, but  $g$  will be determined dynamically.

$$\frac{1}{g^2} = \langle \varphi^{-3/4} \rangle \quad (7)$$

consistent with the expectations in string theory. We have to be aware of the fact that the coupling constant as determined by this naive approximation might be different from that determined by string theory. This approximation is probably only useful in defining the important interactions at low energies. In order to ask more fundamental questions, like the determination of the fundamental coupling constants, the approximation probably has to be improved. This can already be seen when we discuss compactification. One possible way is to compactify on a six-torus  $T^6$ , leading to  $N = 4$  supergravity in  $d = 4$ , which does not resemble known  $d = 4$  phenomenology. One might therefore ask the question for more non-trivial compactifications (still postponing the question of why these should be more likely than the trivial ones). Defining  $\phi = (3/4) \log \varphi$  and neglecting fermionic terms, the equation of motion for  $\phi$  is:

$$\square \phi = \exp(-\phi) \left[ F_{MN}^2 + \exp(-\phi) H_{MNP}^2 \right] \quad (8)$$

Integrating  $\square \phi$  over a compact manifold without boundary leads to a vanishing result. The right-hand side is positive definite and therefore

has to vanish. This implies trivial compactification unless  $\phi \rightarrow \infty$ , which is outside the validity of our approximation. The addition of  $\omega^L$  in  $H$  does not change the situation, but this term requires supersymmetric completion which necessitates the presence of  $R^2$  terms. They actually appear in the Euler combination

$$-\exp(-\phi) \left[ R_{MNPQ}^2 - 4R_{MN}^2 + R^2 \right] \quad (9)$$

on the right-hand side of (8), ensuring the absence of ghosts. With these terms from the  $\alpha'$  expansion, non-trivial compactification is possible:  $R^2$  can be compensated by  $F^2$ , and this implies a breakdown of gauge symmetries in the presence of compactification [14]. Notice, however, that the scale of compactification is not yet fixed. There exists an independent argument confirming this result. For the  $H$  field to be well defined, the integral of the curl of  $H$  over a compact manifold without boundary should vanish:

$$\int_{\tilde{C}_4} dH = \int_{\tilde{C}_4} [Tr F \wedge F - Tr R \wedge R] = 0 \quad (10)$$

leading to a compensation of  $F$  and  $R$  in extra dimensions. These results are very encouraging. If  $E \times E_8$  or  $0(32)$  were to remain unbroken in  $d = 4$ , they would not be able to lead to chiral fermions. The discussed constraints involve integrated quantities and could have various solutions. Only the simplest possibility – a vanishing integrand – can be studied easily [114]. It implies a direct identification of  $F$  and  $R$ . The spin connection  $\omega_m^{ab}$  ( $m = 4, \dots, 9$ ,  $a, b = 1, \dots, 6$ ) can be viewed as a gauge field of an  $0(6)$  subgroup of the Lorentz group  $0(9, 1)$ , identified with  $A_m^A$  in an  $0(6)$  subgroup of  $E_8 \times E_8$  or  $0(32)$  in order to fulfil the constraints. The question of a remaining supersymmetry in  $d = 4$  is related to the holonomy group of the compact manifold, which in turn is a subgroup of  $0(6)$ . I shall not explain this relation here in detail, but just give a heuristic argument. The gravitino  $\psi_M^\alpha$  transforms like a 4 of  $0(6)$ .  $N = 1$  supersymmetry will be present in  $d = 4$  if the decomposition of the 4 with respect to the holonomy group contains exactly one singlet. If there are more singlets, one will have extended supersymmetries, e.g., in the case of the torus the holonomy group is trivial and  $4 = 1 + 1 + 1 + 1$ , resulting in  $N = 4$  supersymmetry. The simplest choice for  $N = 1$  is to have  $SU(3)$  holonomy, which leads to  $4 = 1 + 3$  and  $6 = 3 + \bar{3}$ , and is used in the Calabi-Yau approach. But there are certainly more possibilities, even with discrete subgroups of  $SU(3)$  corresponding to certain orbifolds. For simplicity, I shall here assume  $SU(3)$  holonomy. With this identification of  $\omega$  and  $A$  at least one  $SU(3)$  subgroup of  $0(32)$  or  $E_8 \times E_8$

will break down during compactification. In the case of  $0(32)$ , this will lead to  $0(26) \times U(1)$  with possible zero modes in the decomposition of the adjoint of  $0(32)$ , giving exclusively real representations of  $0(32)$ . Based on this argument, one usually concludes that  $0(32)$  will not lead to a phenomenologically successful model, although not all possibilities have yet been studied. The situation in the case of  $E_8 \times E_8$  looks better. A decomposition of the adjoint of  $E_8$  with respect to  $E_6 \times SU(3)$  leads to  $248 = (78, 1) + (27, 3) + (\bar{27}, \bar{3}) + (1, 8)$  and contains chiral representations. Moreover,  $E_6$  is one of the more successful candidates for a grand unified gauge group with a family of quarks and leptons in 27, the number of these zero modes being defined by topological properties of the compact manifold. Here is then a common starting point for the construction of "superstring-inspired models".

### 3. Towards $d = 4$

We have first to discuss the possible zero modes. Let us define indices  $M = (\mu, m)$  ( $\mu = 0, \dots, 3$ ;  $m = 4, \dots, 9$ ) and start with the metric

$$g_{MN} = \left( \begin{array}{c|c} g_6^{-1/2} \hat{g}_{\mu\nu} & \\ \hline & g_{mn} \end{array} \right) \quad (11)$$

where  $g_6 = \det g_{mn}$  is used to redefine  $g_{\mu\nu}$  in order to have usual kinetic terms for the graviton. The integral over extra dimensions

$$\int d^6 y \sqrt{-g_6} = R_c^6 \sim \frac{1}{M_c^6} \quad (12)$$

defines the average radius of compactification. Defining  $g_{mn} = \exp(\sigma) \hat{g}_{mn}$ , one can then normalize  $\int d^6 \sqrt{-\hat{g}_6} = M_P^{-6}$  and  $\exp(\sigma)$  defines the radius of compactification in units of the Planck length. Depending on the topological properties of the manifold,  $g_{mn}$  gives rise to zero modes that are scalars in  $d = 4$  (we will not discuss off-diagonal terms in  $g_{MN}$  like  $g_{\mu m}$  that give rise to gauge bosons depending on the isometries of the manifold).  $g_{mn}$  corresponds to a symmetric tensor of  $0(6)$  with respect to the  $SU(3)$  subgroup discussed earlier; we have  $21 = 1 + 8 + 6 + \bar{6}$ . With the notation  $m = (i, \bar{j})$ , the latter correspond to modes of  $g_{i\bar{j}}, g_{ij}, g_{\bar{i}\bar{j}}$ , while  $\sigma$  is the singlet.

Turning to the gravitino  $\psi_M^\alpha$ , we can view  $\alpha$  as an eight-dimensional index which transforms as a 4 of  $0(6)$  and a Weyl spinor of  $0(3, 1)$ .  $\psi_\mu^\alpha$  corresponds to spin-3/2 particles in  $d = 4$  with  $N_{max} = 4$  as already discussed.  $\psi_m^\alpha$  can give rise to spin- $\frac{1}{2}$  zero modes. To obtain canonical kinetic terms for the gravitino, as in the case of the metric, a rescaling

$$\tilde{\psi}_\mu = \exp(-3\sigma/4) \psi_\mu \quad (13)$$

is required.

The antisymmetric tensor field  $B_{MN}$  could give rise to  $B_{\mu\nu}$ ,  $B_{m\nu}$  and  $B_{mn}$  (corresponding to the Betti numbers  $b_0, b_1$  and  $b_2$ ). A zero mode from  $B_{\mu\nu}$  corresponds to one pseudoscalar degree of freedom  $\theta$  defined through a duality transformation

$$H_{\mu\nu\rho}\epsilon^{\mu\nu\rho\sigma} = \varphi^{3/2} \exp(-6\sigma) \partial^\sigma \theta + \dots \quad (14)$$

$B_{m\nu}$  could give rise to extra gauge bosons which (although possibly interesting) we shall not discuss here.  $B_{mn}$  will again correspond to pseudoscalars in  $d = 4$ . A decomposition with respect to  $SU(3)$  gives  $15 = 1 + 3 + \bar{3} + 8$  with the singlet corresponding to the "trace"  $\eta = \epsilon^{mn} B_{mn}$  and  $B_{\bar{i}\bar{j}}$ , and  $B_{ij}$  and  $B_{\bar{i}\bar{j}}$  corresponding to  $3, \bar{3}$ , and  $8$  respectively. All these modes appear in the action only through the field strength  $H$  implying derivative couplings, i.e., they show axion-like behaviour. From the  $\lambda, \phi$  members of the supergravity multiplet, we expect additional spin  $-\frac{1}{2}(0)$  particles in  $d = 4$ .

The discussion of the zero modes of  $A_M^A$  involves some complication because of the identification of  $\omega_m^{ab}$  and  $A_m^A$  in an  $SU(3)$  subgroup.  $A_\mu^A$  will of course, give rise to gauge bosons in the adjoint representations of the unbroken gauge group, e.g.,  $A = 1, \dots, 78$  for  $E_6$ .  $A_m^A$  will give rise to scalars in  $d = 4$ , and we are mostly interested in those transforming as  $27$  (or  $\bar{27}$ ) under  $E_6$ . Let us therefore write  $A = (a, i)$  or  $(\bar{a}, \bar{i})$   $a = 1, \dots, 27$ . The states  $C^b = A_i^{b,i}$  and  $B^{\bar{b}} = A_{\bar{i}}^{\bar{b},\bar{i}}$  then transform as  $27, \bar{27}$  with respect to  $E_6$  and are singlets under the diagonal subgroup  $SU(3)$  of the product of  $SU(3) \subset O(6)$  and  $SU(3) \subset E_8$ . These bosons will have supersymmetric partners from the zero modes of  $\chi_\alpha^A$ . The number of the possible zero models is of course entirely defined by the topological properties of the manifold under consideration.

We can now have a first look at the possible interactions of these zero models in  $d = 4$  starting from the  $d = 10$  action given in (1). Of course, in general we expect here not only the influence of topological properties, but also the explicit form of the metric of the compact manifold will become important. Nonetheless we will be able to obtain some non-trivial results that are rather independent of the special form of the metric. We will do that exclusively in the framework of  $N = 1$  supergravity in  $d = 4$ , firstly because of the reasons given in Section 2, and secondly because this theory is simpler than the non-supersymmetric case.

$N = 1$  supergravity in  $d = 4$  (with action including terms up to two derivatives [21]) is defined through two functions of the chiral superfields  $\phi_i$ . The first is an analytic function  $f(\phi_i)$  defining the gauge kinetic terms  $f(\phi_i)W^\alpha W_\alpha$ . In a component language,  $f$  appears in many places,

but it can be extracted most efficiently from

$$\text{Ref}(\varphi_i)F_{\mu\nu}F^{\mu\nu} + \text{Imf}(\varphi_i)\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} \quad (15)$$

where  $\varphi_i$  denotes the (complex) scalar component of  $\phi_i$ . The second is the so-called Kähler potential

$$G(\phi_i, \phi_i^*) = K(\phi_i, \phi_i^*) + \log|W(\phi_i)|^2 \quad (16)$$

Unlike  $f$ ,  $G$  is not analytic and contains the left-handed chiral superfields along with their complex conjugates. The second term in (16) contains the analytic function  $W(\phi_i)$ : the superpotential. The action in component form usually contains  $G$  in complicated form; the scalar kinetic terms, e.g., are

$$G_i^j(\partial_\mu\varphi^i)(\partial^\mu\varphi_j^*); \quad G_i^j \equiv \frac{\partial^2 G}{\partial\varphi^i\partial\varphi_j^*} \quad (17)$$

whereas the scalar potential is given by

$$V = \exp(G)[G_k(G^{-1})^k_l G^l - 3] \quad (18)$$

which makes it difficult to extract  $G$  once an action is given in component form. There is only one term which allows a rather simple identification of  $G$ , and this is a term involving the gravitino

$$e_4 \exp(G/2) \bar{\psi}_\mu \gamma^{\mu\nu} \gamma_5 \psi_\nu \quad (19)$$

which will later be used extensively after the correct redefinitions of the gravitino in  $d = 4$  have been performed. Let us now consider the action in  $d = 10$  in order to learn something about the possible action in  $d = 4$ . We start with the gauge kinetic term

$$e_{10}\varphi^{-3/4}F_{MN}F^{MN} \quad (20)$$

Since we are interested in the  $F_{\mu\nu}^2$  part, we write

$$e_4 e_6 \varphi^{-3/4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \quad (21)$$

where, with the definitions given earlier, we would like to extract  $f$  from

$$\hat{e}_4 \text{Ref} F_{\mu\nu} F^{\mu\nu} \quad (22)$$

with  $\hat{e}_4 = (\det \hat{g}_{\mu\nu})^{\frac{1}{2}} = \exp(6\sigma)e_4$ , and indices are contracted with the "hatted" metric. Integrating the extra six dimensions with the normalization given in (13) using  $M_P \equiv 1$ , we obtain

$$ReS \equiv Ref = \varphi^{-3/4} \exp(3\sigma) \quad (23)$$



as the real part of the scalar component of a chiral superfield denoted by  $S$ . This is a rather amazing result. Remember that at no point in the derivation did we have to know something about the metric of the compact six-dimensional space, so this constitutes a rather model-independent result. Observe that  $f$  is usually non-trivial, that its vacuum expectation value (vev) will determine the gauge coupling constant, and that the couplings of  $E_8$  (or  $E_6$ ) and  $E'_8$  coincide.

Let us now discuss the imaginary part of  $f$ , to be extracted from  $F_{\mu\nu}F_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}$ . The relevant degree of freedom comes from  $B_{\mu\nu}$  as discussed earlier.  $B_{\mu\nu}$  couples only through its field strength  $H_{\mu\nu\rho}$  and has therefore only derivative couplings. Taking the relevant terms in the  $d = 10$  action and integrating the extra dimensions, we obtain

$$\varphi^{-3/2} \exp(6\sigma) H_{\mu\nu\rho} H^{\mu\nu\rho} + H_{\mu\nu\rho} O^{\mu\nu\rho} \quad (24)$$

where  $O^{\mu\nu\rho}$  contains fermion bilinears.  $H$  has to satisfy a constraint (neglecting  $R^2$ -terms for the moment)

$$\partial_{[\mu} H_{\nu\rho\sigma]} = -Tr F_{[\mu\nu} F_{\rho\sigma]} \quad (25)$$

which we take into account by adding a Lagrange multiplier

$$\theta \epsilon^{\mu\nu\rho\sigma} (\partial_\mu H_{\nu\rho\sigma} + Tr F_{\mu\nu} F_{\rho\sigma}) \quad (26)$$

Next we eliminate  $H$  via the equations of motion and arrive at an action containing the terms

$$\varphi^{3/2} \exp(-6\sigma) (\partial_\mu \theta)^2 + \theta \epsilon^{\mu\nu\rho\sigma} Tr(F_{\mu\nu} F_{\rho\sigma}) \quad (27)$$

which tells us that  $Im f = \theta$ , and for the scalar component of  $S$  we obtain

$$S = \varphi^{-3/4} \exp(+3\sigma) + i\theta \quad (28)$$

as a mixture of  $g_{MN}$  and  $B_{MN}$  zero modes. The partner is a combination of  $\psi_m$  and  $\lambda$  zero modes which we will not discuss here in detail. Observe that  $\theta$  couples only with derivatives except for the last term in (27), and that the  $d = 4$  action, has a Peccei-Quinn-like symmetry under shifts of  $\theta$  by a real constant, thus  $\theta$  couples like an axion. Let me stress again that all these statements about the action and the form of (28) are model-independent and could be derived without explicit knowledge of the metric.

Unfortunately, the situation changes once we try to extract the Kähler potential. As already indicated, the term to investigate is the  $d = 4$  "gravitino mass term" (19). The extraction of this term is rather complicated due to several redefinitions of the gravitino field. A general form

has been given in [29]), and we will not repeat the derivation here. Many of the terms appearing there depend explicitly on the metric and spin-connection of the six-dimensional compact space. A model-independent statement can only be made about the structure of the superpotential, because it is an analytic function in the chiral superfields. Symbolically the "gravitino mass term" is obtained as

$$\exp(G/2) = \varphi^{-3/4} \exp(-3\sigma) \Gamma^{mnp} H_{mnp} \quad (29)$$

and from (16) we can try to read off the superpotential.  $W(\phi_i)$  is defined to be an analytic function in the chiral superfields and should not contain derivatives. A first inspection of (29) therefore suggests that a possible candidate for a superpotential is the  $A^3$  term contained in the Yang-Mills Chern-Simons term (4) included in  $H$ . This then gives rise to a trilinear superpotential involving the  $C$  and  $B$  fields defined earlier. At the moment it is not clear whether these are the only possible terms in the superpotential, although at the classical level this seems to be the complete expression. Observe that, for example, the superfield  $S$  as defined in (28) cannot appear in the superpotential, since its pseudoscalar component has only derivative couplings. We will come back to these points later. In any case, a more detailed discussion of the Kähler potential requires more information (or approximations) about the  $d = 6$  metric. Before we tackle this topic, let me first present a discussion about supersymmetry breakdown in  $d = 4$ .

#### 4. Gaugino condensation and supersymmetry breakdown

$N = 1$  supergravity in  $d = 4$  still needs the incorporation of supersymmetry breakdown at a scale small compared to the Planck mass. For the phenomenological reasons mentioned earlier, this should appear in a hidden sector only coupled gravitationally to the observable sector. Some superstring models contain such a hidden sector, e.g. the sector that contains the particles transforming non-trivially under the second  $E_8$ . Notice that the observable sector (for definiteness called the  $E_6$  sector) only couples gravitationally to the  $E'_8$  sector (there are no particles that transform non-trivially both under  $E_6$  and  $E'_8$ ). Moreover, the  $E'_8$  sector contains a  $d = 10$  pure super-Yang-Mills multiplet, suggesting a possible breakdown of supersymmetry via gaugino condensates. This breakdown has already been discussed in the framework of supergravity models, both at the level of an effective Lagrangian [92] and at the level of the complete classical action [40]. Assume asymptotically-free gauge

interactions (here  $E'_8$  or a subgroup thereof) with a scale

$$\Lambda = \mu \exp \left( -1/b_0 g^2(\mu) \right) \quad (30)$$

which is renormalization-group invariant at the level of the one-loop  $\beta$ -function. In analogy to QCD, which leads to  $q\bar{q}$  condensates, we will here assume that the gauge fermions condense at a scale

$$\langle \chi\chi \rangle = \Lambda^3 \quad (31)$$

As long as  $\Lambda$  is small compared to  $M_p$ , we assume that gravity will not qualitatively disturb this dynamical mechanism. The question whether such a condensate breaks supersymmetry can be studied by investigating the supersymmetry transformation laws of the fermionic fields of the theory. The non-derivative terms in these transformations will give us the auxiliary fields that serve as order parameters for supersymmetry breakdown. The relevant objects here are the auxiliary fields of the chiral superfields

$$F_k = \exp(G/2)G_k - \frac{1}{4}f_k(\chi\chi) + \dots \quad (32)$$

where  $f$  is the gauge kinetic function discussed earlier and  $f_k$  is its derivative with respect to  $\phi_k$ . A necessary condition for the breakdown of supersymmetry via gaugino condensates is therefore a non-trivial  $f$ -function. This condition is fulfilled in the framework of superstring-inspired models [28] [32], since we have seen in the last section that  $f = S$  in a rather model-independent way. Whether this is also sufficient for the breakdown of supersymmetry can only be checked by minimizing the potential

$$V = F_k \left( G^{-1} \right)_\ell^k F^\ell - 3 \exp(G) \quad (33)$$

since the different terms in (32) might cancel at the minimum. But let us for the moment assume that only the second term in (32) receives a *vev*. Since  $f_s = 1$  in units of  $M_p$ , we find a supersymmetry breakdown scale

$$\langle F_s \rangle = M_s^2 \approx \Lambda^3/M_p \quad (34)$$

and a scale of  $\Lambda \sim 10^{13}$  GeV would lead to a gravitino mass in the TeV range. Once we understand why  $\Lambda$  is five orders of magnitude smaller than  $M_p$ , we shall understand why  $m_{3/2}/M_p \sim 10^{-15}$ .  $\Lambda$  now depends on the  $E'_8$  gauge coupling and the spectrum of low-energy modes. Identifying  $g_6$  with  $g_8$  would in many circumstances lead to too large a value for  $\Lambda$ , and one might speculate that  $E'_8$  should break during compactification. We shall, however, see later that the equality of  $g_6$  and  $g_8$  seems

to be only an artifact of the classical approximation, which is not true in the full theory. Thus the shadow  $E'_8$  (or a subgroup thereof) sector of the superstring takes the role of the hidden sector of supergravity models and might explain the smallness of  $m_{3/2}$  compared to  $M_p$ . But how does this breakdown of SUSY in the hidden sector influence the observable sector? In general, we would expect gaugino masses ( $m_{1/2}$ ), scalar masses ( $m_0$ ) and the trilinear couplings ( $Am$ ) to be of the order of magnitude of  $m_{3/2}$ . A naive inspection shows that this might also be true here. Gaugino masses in the observables sector are in general given by

$$m_{1/2} = f_k \left( G^{-1} \right)_\ell^k F^\ell \quad (35)$$

where  $f$  is the gauge kinetic function of the observable sector. With  $F^\ell = (1/4)f^\ell < \chi\chi >$  we would therefore obtain  $m_{1/2} \sim m_{3/2}$ . In the same way we would obtain under these circumstances the soft trilinear couplings  $A \sim 1$  and scalar masses of order  $m_{3/2}$ . To make a quantitative statement about the soft parameters we need a better understanding of the Kähler potential, a question which we want to discuss in the next section. We need this in order to study the explicit form of the effective potential and find its minimum. This then also has to determine the exact value of  $\Lambda$  in (30) which depends on the coupling constant and is fixed only after the value of  $g$  is known.

## 5. Reduction and Truncation

A first approximation for  $G$  (that might simulate an orbifold approximation of interest in this context) is obtained through reduction and truncation [118]. One first compactifies the  $d = 10$  theory on a six-torus  $T^6$ . The resulting theory is  $N = 4$  supersymmetric in  $d = 4$ . From this theory one truncates unwanted states, to obtain an  $N = 1$  theory. From the gauge singlet sector one keeps only those states that transform as singlets under an  $SU(3) \subset 0(6)$  of the Lorentz group. Since  $\psi_\mu^\alpha$  transforms as a 4 of  $0(6)$  and thus as  $1 + 3$  under  $SU(3)$ , we remain with one gravitino. As already explained in Section 3, there are only a few gauge singlets that survive this truncation. For the bosonic modes we have  $\varphi$ ,  $\sigma$  from the metric as well as  $\theta$  and  $\eta$  from the antisymmetric tensor. For the gauge non-singlet fields one has to remember the identification of spin-connection and gauge fields. Here one keeps those states which are singlets under the diagonal subgroup of the product of  $SU(3) \subset 0(6)$  and  $SU(3) \subset E_8$ . This leaves us with one 27 of  $E_6$  in this case, corresponding to  $C^b = A_i^{b,i}$ ; ( $b = 1, \dots, 27$ , cf. Section 3). With this well-defined procedure based on simple reduction on  $T^6$ , the component Lagrangian

in  $d = 4$  can be deduced. From this we can immediately read off  $f = S$  and  $W = d_{abc}C^aC^bC^c$ , which should not be surprising. Moreover, from the "gravitino mass term" formula (29) one obtains

$$G = \log \left( e^{-6\sigma} \varphi^{-3/2} \right) + \log |W|^2 \quad (36)$$

The components  $\varphi$  and  $\sigma$  should correspond to lowest components of chiral superfields. One combination  $S = \varphi^{-3/4} \exp(3\sigma) + i\theta$  has already been defined earlier. To define the other combination, the information from (36) is not enough. The charged fields  $C$  do not yet appear in the first term of  $G$  in (36) and the correct definition of the superfields has yet to be found. This can be done, for example, by using the scalar kinetic terms. It leads to a second superfield in which  $\varphi, \sigma$  and the  $C$ -modes mix

$$T = \exp(\sigma) \varphi^{3/4} + |C_a|^2 + i\eta \quad (37)$$

where  $\eta$  is the mode from  $\epsilon^{mn}B_{mn}$  as discussed earlier, and the Kähler potential from (36) thus reads

$$G = -\log(S + S^*) - 3\log(T + T^* - 2|C|^2) + \log |W|^2 \quad (38)$$

a form already previously mentioned in the framework of supergravity models. The scalar potential derived from this  $G$ -function has some remarkable properties

$$V = \frac{1}{16st_c^3} \left[ |W|^2 + \frac{t_c}{3} |W'|^2 \right] + D^2 - \text{terms} \quad (39)$$

where  $s = ReS$  and  $t_c = ReT - |C_a|^2 = t - |C_a|^2$  and  $W'$  is the derivative of  $W$  with respect to the  $C$ -field. The potential is positive definite ( $t_c > 0$  is required by the kinetic terms) and has a minimum with vanishing vacuum energy  $V = 0$ . This minimum is obtained at  $W = W' = 0$  independent of the values of  $s$  and  $t$ . This implies that at this level the gauge coupling constant and the radius of compactification is not yet fixed. The theory has classical symmetries which allow shifts of the values of  $s$  and  $t$ , as well as Peccei-Quinn symmetries corresponding to shifts in  $\theta$  and  $\eta$ . This, of course, makes the use of this approximation as an effective low-energy limit of the superstring very problematic. Certain crucial parameters, like the value of the gauge coupling constant and the scale of compactification, which we believe to be dynamically determined in the full string theory, are not yet fixed. To determine these quantities we would need information beyond the truncated theory.

This remains a relevant question when we discuss the effective potential in the presence of a gaugino condensate. Since the gauge coupling

constant is not determined,  $\Lambda$  in (30) is also unknown. Using (33) and (38), we get for the potential

$$V = \frac{1}{16st_c^3} \left[ |W - 2(st_c)^{3/2}(\chi\chi)|^2 + \frac{t_c}{3}|W'|^2 \right] \quad (40)$$

where  $(\chi\chi)$  depends on  $g^2$  through  $\exp(-S/b_0)$ . The potential is still positive definite and has a minimum at  $V = 0$  which is still degenerate. Now the minimum need not necessarily imply  $W = W' = 0$ , but we could have a non-trivial vev of  $W$ . Given fixed  $\langle W \rangle \neq 0$  by some yet unknown mechanism, the value of the gauge coupling constant would be fixed. The most natural candidate for such a mechanism would be a non-trivial vev (so-called flux) of the antisymmetric tensor field  $H$  as defined in (5). At first sight one might have conjectured that the appropriate flux would originate from  $dB$ , but it was soon realized that  $\langle dB \rangle$  is quantized in units of the Planck scale [106]. Thus  $\langle dB \rangle$  should vanish in order to allow for a value of  $\langle \chi\chi \rangle$  that is small enough to give a reasonable value for the gravitino mass after supersymmetry breakdown. One therefore concluded that it is the flux of the Chern-Simons terms in (5) which is responsible for the non-zero  $H$ -flux [29]. For these terms the quantization argument of ref. [106] does not apply and acceptable values for  $\langle \chi\chi \rangle$  can be obtained. We shall come back to this question in more detail in the framework of the heterotic M-theory. There it will become obvious from theoretical arguments that it is not  $\langle dB \rangle$  but the flux from the Chern-Simons terms that compensates the contribution of the gaugino condensate in the effective potential.

In order to minimize the potential, the theory slides to a coupling constant which, through (30), gives a value of the condensate that exactly cancels the contribution of  $W$ . In other words, this means that the dilaton  $S$  slides to a value that cancels the vacuum energy in the same way as an axion slides to cancel a possible  $\theta$ -parameter of a gauge theory [observe that  $\exp(-S/b_0)$  contains both  $s$  and  $\theta$ ]. Although we do not yet understand the magnitude of supersymmetry breakdown, this mechanism to ensure  $E_{\text{vacuum}} = 0$  after  $SUSY$  breakdown appears very attractive. We shall still need to convince ourselves that supersymmetry is actually broken, since in (40) a certain cancellation of  $\langle W \rangle$  and  $\langle \chi\chi \rangle$  appears. In fact it tells us that the auxiliary field  $F_S$  of the  $S$ -superfield vanishes in the vacuum. Nonetheless, here  $F_T$  requires a non-vanishing vev once  $\langle W \rangle \neq 0$ , and supersymmetry is broken

$$F_T = \exp(G/2)G_T \neq 0. \quad (41)$$

In a next step we have to analyze how the breakdown of  $SUSY$  is felt in the observable sector, and this is, of course, model dependent.

Gaugino masses, for example, are given by  $m_{1/2} = f_k(G^{-1})_\ell^k F^\ell$ , and only  $f_S$  is different from zero. In the case at hand we therefore obtain  $m_{1/2} = 0$ . In fact, the same is also true for the scalar masses. This is an artifact of the special model (a so-called no scale model) and would need further discussions. A first question concerns the stability of this result in perturbation theory which we shall investigate in the next section.

Before we do this let us mention a different way to include the gaugino condensate in the low-energy effective potential [32]. Instead of including  $(\chi\chi)$  in  $F_k$  directly, as in (32), one might postulate a new contribution to the superpotential proportional to  $(\chi\chi) \sim \exp(-S/b_0)$ . This leads to a potential very similar although not identical to the one given in (40) (for details see [32, 29]).

## 6. Beyond the classical level

Vanishing values of the soft parameters in the observable sector might be a result of the symmetries of the theory, and if yes, whether these symmetries hold to all orders in the perturbative loop expansion. In the heterotic string this loop expansion is governed by the coupling constant  $g$ , which in turn is defined through a *vev* of the dilation field. This will allow us to construct a definite loop expansion in the dilation field and still give us restrictions on how the classical symmetries are broken by loop effects. But before we discuss the loop expansion in more general terms, let us examine some aspects at the one-loop level. We can do that because of the mechanism of anomaly cancellation in the  $d = 10$  field theory. Green and Schwarz have observed that the cancellation of anomalies [49] requires certain new local counterterms with definite finite coefficients in the one-loop effective action to cancel the gauge non-invariance of present non-local terms. In general, such terms appear with infinite coefficients, but the possible symmetry of the effective action forces us to renormalize the theory in such a way that these gauge-variant local counterterms have a well-defined finite coefficient. An example of such a term is

$$\epsilon B_{VO} Tr(F_{LK} F_{SW}) Tr(F_{AG} F_{EN}) \epsilon^{VOLKSWAGEN} \quad (42)$$

where  $\epsilon = 1/720(2\pi)^5$ . While this gives rise to many new interaction terms in the  $d = 4$  theory, one possible manifestation seems to be of particular importance. Replacing one of the  $Tr F^2$  terms by their vev in extra dimensions, one arrives at

$$\eta \epsilon^{\mu\nu\rho\sigma} Tr(F_{\mu\nu} F_{\rho\sigma}) \quad (43)$$

$\eta$  is the imaginary part of  $T$ , and unlike in the classical case it now (in addition to  $\theta$ ) couples to  $F\tilde{F}$ . Observe that (43) is gauge-invariant, while

(42) is not, but is required by the absence of anomalies in  $d = 10$ . This shows that the remnants of such terms originate in ten dimensions, and are one of the few places where we could in principle observe whether we live in higher dimensions. (43) suggests that not only  $\theta$ , but also  $\eta$ , couples like an axion. To make sure that this does not lead just to a redefinition of  $\theta$  at the one-loop level, all anomaly cancellation terms have to be considered. Doing this and satisfying  $Tr F^2 = Tr R^2$  in extra dimensions, one arrives at the result that  $\eta$  couples differently to  $E_6$  and  $E'_8$ :

$$\epsilon\eta[(F\tilde{F})_8 - (F\tilde{F})_6] \quad \text{and} \quad \theta[(F\tilde{F})_8 + (F\tilde{F})_6] \quad (44)$$

leading to different gauge kinetic functions

$$f = S \pm \epsilon T \quad (45)$$

for the different gauge groups.

This fact has interesting consequences, some of which we will now list.

- a) The second axion could be a candidate to solve the strong CP problem of QCD in the observable sector. One axion (like  $\theta$  alone) would not be sufficient, because it is used to adjust the  $\theta$ -angle of  $E'_8$  and becomes massive. For a relatively recent discussion see [43].
- b) Supersymmetry requires the same behaviour of the real parts of  $S$  and  $T$  as that of the imaginary part; i.e.,  $ReS$  and  $ReT$  couple differently to  $E_6$  and  $E'_8$ . Since the vevs of these fields define the gauge coupling constants,  $g_6$  and  $g'_8$  need no longer be equal. This might have consequences for the condensation scale of  $E'_8$ .
- c) There exist now two axion-dilaton pairs, and this might generalize the relaxation of the cosmological constant to the observable sector in the same way as it appears in the hidden sector [70].
- d) Imposition of supersymmetry also requires new terms in the Kähler potential at the one-loop level. We will discuss this later.
- e) As expected, these effects at the one-loop level lead to an induced breakdown of supersymmetry in the observable sector once it is broken in the hidden sector. Remember our discussion in Section 5, where the observable sector remained supersymmetric. Gaugino masses are given by

$$m_{1/2} \sim F_T f_T + F_S f_S. \quad (46)$$

At tree level we had  $F_S = f_T = 0$  and vanishing gaugino masses. But now we have  $f = S + \epsilon T$ , and  $f_T$  no longer vanishes. As



a result, non-trivial gaugino masses (and also non-trivial scalar masses and A-parameters) of order  $\epsilon m_{3/2}$  are transmitted to the observable sector.

Of course, in case of a nontrivial  $\epsilon$ , we should go back to the action and see what happens to the one-loop effective potential. In general we shall expect a nontrivial value of the cosmological constant (typically anti-de Sitter) and in some cases even unbroken supersymmetry. So a discussion of a fully realistic model needs more structure than present in the toy model under consideration.

## 7. Heterotic M-Theory

With the discovery of string dualities, there has been a revival of the study of those string theories that might eventually become relevant for our discussion of the low-energy effective supergravity theories. From all the new and interesting results in string dualities, it is the heterotic M-theory of Hořava and Witten [53] (that in  $d = 11$  could be regarded as the strong coupling limit of  $d = 10$   $E_8 \times E_8$  heterotic string theory) which might have a direct impact on the discussion of the phenomenological aspects of these theories. One of the results concerns the question of the unification of all fundamental coupling constants [115] and the second one the properties of the soft terms (especially the gaugino masses) once supersymmetry is broken [101, 102]. As we shall see in both cases, results that appear problematic in the weakly coupled case (as the formerly discussed heterotic string case will be called from now on) get modified in a satisfactory way, while the overall qualitative picture remains essentially unchanged. In these lectures we shall therefore concentrate on these aspects of the new picture.

The heterotic M-theory is an 11-dimensional theory with the  $E_8 \times E_8$  gauge fields living on two 10-dimensional boundaries (walls), respectively, while the gravitational fields can propagate in the bulk as well. A  $d = 4$  dimensional theory with  $N = 1$  supersymmetry emerges at low energies when 6 dimensions are compactified on a Calabi-Yau manifold. The scales of that theory are  $M_{11}$ , the  $d = 11$  Planck scale,  $R_{11}$  the size of the  $x^{11}$  interval, and  $V \sim R^6$  the volume of the Calabi-Yau manifold. The quantities of interest in  $d = 4$ , the Planck mass, the GUT-scale and the unified gauge coupling constant  $\alpha_{GUT}$  should be determined through these higher dimensional quantities. The fit of ref. [115] identifies  $M_{GUT} \sim 3 \cdot 10^{16}$  GeV with the inverse Calabi-Yau radius  $R^{-1}$ . Adjusting  $\alpha_{GUT} = 1/25$  gives  $M_{11}$  to be a few times larger than  $M_{GUT}$ . On the other hand, the fit of the actual value of the Planck scale can be achieved by the choice of  $R_{11}$  and, interestingly enough,  $R_{11}$  turns out

to be an order of magnitude larger than the fundamental length scale  $M_{11}^{-1}$ . A satisfactory fit of the  $d = 4$  scales is thus possible, in contrast to the case of the weakly coupled heterotic string where, naively, the string scale seems to be a factor 20 larger than  $M_{GUT}$ .

### 7.1 The action in $d = 11$

The effective action of the strongly coupled  $E_8 \times E_8 - M$ -theory in the “downstairs” approach is given by [53] (we take into account the numerical corrections found in [20])

$$\begin{aligned}
L = & \frac{1}{\kappa^2} \int d^{11}x \sqrt{g} \left[ -\frac{1}{2}R - \frac{1}{2}\bar{\psi}_I \Gamma^{IJK} D_J \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} \right. \\
& - \frac{\sqrt{2}}{384} \left( \bar{\psi}_I \Gamma^{IJKLMN} \psi_N + 12 \bar{\psi}^J \Gamma^{KL} \psi^M \right) \left( G_{JKLM} + \hat{G}_{JKLM} \right) \\
& \left. - \frac{\sqrt{2}}{3456} \epsilon^{I_1 I_2 \dots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{11}} \right] \\
& + \frac{1}{4\pi(4\pi\kappa^2)^{2/3}} \int_{M_i^{10}} d^{10}x \sqrt{g} \left[ -\frac{1}{4} F_{iAB}^a F_i^{aAB} - \frac{1}{2} \bar{\chi}_i^a \Gamma^A D_A (\hat{\Omega}) \chi_i^a \right. \\
& \left. - \frac{1}{8} \bar{\psi}_A \Gamma^{BC} \Gamma^A \left( F_{iBC}^a + \hat{F}_{iBC}^a \right) \chi_i^a + \frac{\sqrt{2}}{48} \left( \bar{\chi}_i^a \Gamma^{ABC} \chi_i^a \right) \hat{G}_{ABC11} \right]
\end{aligned} \tag{47}$$

where  $M^{11}$  is the  $d = 11$  manifold and  $M_i^{10}$  its 10-dimensional boundaries. In the lowest approximation  $M^{11}$  is just a product  $M^4 \times X^6 \times S^1/Z_2$ . Compactifying to  $d = 4$  in such an approximation we obtain [115, 20]

$$G_N = \frac{\kappa_4^2}{8\pi} = \frac{\kappa^2}{8\pi R_{11} V}, \tag{48}$$

$$\alpha_{GUT} = \frac{(4\pi\kappa^2)^{2/3}}{V} \tag{49}$$

with  $V$  the volume of the Calabi–Yau manifold  $X^6$  and  $R_{11} = \pi\rho$  the  $S^1/Z_2$  length.

The fundamental mass scale of the 11-dimensional theory is given by  $M_{11} = \kappa^{-2/9}$ . Let us see which value of  $M_{11}$  is favoured in a phenomenological application. For that purpose we identify the Calabi–Yau volume  $V$  with the GUT-scale:  $V \sim (M_{GUT})^{-6}$ . From (49) and the value of  $\alpha_{GUT} = 1/25$  at the grand unified scale, we can then deduce the value of  $M_{11}$

$$V^{1/6} M_{11} = (4\pi)^{1/9} \alpha_{GUT}^{-1/6} \approx 2.3, \tag{50}$$

to be a few times larger than the GUT-scale. In a next step we can now adjust the gravitational coupling constant by choosing the appropriate value of  $R_{11}$  using (48). This leads to

$$R_{11}M_{11} = \left(\frac{M_{Planck}}{M_{11}}\right)^2 \frac{\alpha_{GUT}}{8\pi(4\pi)^{2/3}} \approx 2.9 \cdot 10^{-4} \left(\frac{M_{Planck}}{M_{11}}\right)^2. \quad (51)$$

This simple analysis tells us the following:

- In contrast to the weakly coupled case, the correct value of  $M_{Planck}$  can be fitted by adjusting the value of  $R_{11}$ .
- The numerical value of  $R_{11}^{-1}$  turns out to be approximately an order of magnitude smaller than  $M_{11}$ .
- Thus the 11th dimension appears to be larger than the dimensions compactified on the Calabi–Yau manifold, and at an intermediate stage the world appears 5-dimensional with two 4-dimensional boundaries (walls).

We thus have the following picture of the evolution and unification of coupling constants. At low energies the world is 4-dimensional and the couplings evolve accordingly with energy: a logarithmic variation of gauge coupling constants and the usual power law behaviour for the gravitational coupling. Around  $R_{11}^{-1}$  we have an additional 5th dimension and the power law evolution of the gravitational interactions changes. Gauge couplings are not effected at that scale since the gauge fields live on the walls and do not feel the existence of the 5th dimension. Finally at  $M_{GUT}$  the theory becomes 11-dimensional and both gravitational and gauge couplings show a power law behaviour and meet at the scale  $M_{11}$ , the fundamental scale of the theory. It is obvious that the correct choice of  $R_{11}$  is needed to achieve unification. We also see that, although the theory is weakly coupled at  $M_{GUT}$ , this is no longer true at  $M_{11}$ . The naive estimate for the evolution of the gauge coupling constants between  $M_{GUT}$  and  $M_{11}$  goes with the sixth power of the scale. At  $M_{11}$  we thus expect unification of the couplings at  $\alpha \sim O(1)$ . In that sense, the M-theoretic description of the heterotic string gives an interpolation between weak coupling and moderate coupling. In  $d = 4$  this is not strong-weak coupling duality in the usual sense. We shall later come back to these questions when we discuss the appearance of a critical limit on the size of  $R_{11}$ . A value of  $\alpha \sim O(1)$  (and thus  $S \sim O(1)$ ) at  $M_{11}$  might also be favoured in view of the question of the dynamical determination of the vev of the dilaton field [72].

## 7.2 The effective action in $d = 4$

We now want to perform a compactification from  $d = 11$  to  $d = 4$ . Again we use the method of reduction and truncation. For the metric we write

$$g_{MN}^{(11)} = \begin{pmatrix} c_4 e^{-\gamma} e^{-2\sigma} g_{\mu\nu} & & \\ & e^\sigma g_{mn} & \\ & & e^{2\gamma} e^{-2\sigma} \end{pmatrix} \quad (52)$$

with  $M, N = 1 \dots 11$ ;  $\mu, \nu = 1 \dots 4$ ;  $m, n = 5 \dots 10$  and  $\det(g_{mn})=1$ . This is the frame in which the 11-dimensional Einstein action gives the ordinary Einstein action after the reduction to  $d = 4$ :

$$-\frac{1}{2\kappa^2} \int d^{11}x \sqrt{g^{(11)}} R^{(11)} = -\frac{c_4 \hat{V}_7}{2\kappa^2} \int d^4x \sqrt{g} R + \dots \quad (53)$$

where  $\hat{V}_7 = \int d^7x$  is the coordinate volume of the compact 7-manifold and the scaling factor  $c_4$  describes our freedom to choose the units in  $d = 4$ . The most popular choice in the literature is  $c_4 = 1$ . This, however, corresponds to the unphysical situation in which the 4-dimensional Planck mass is determined by the choice of  $\hat{V}_7$  which is just a convention. With  $c_4 = 1$  one needs further rescaling of the 4-dimensional metric. We instead prefer the choice

$$c_4 = V_7 / \hat{V}_7 \quad (54)$$

where  $V_7 = \int d^7x \sqrt{g^{(7)}}$  is the physical volume of the compact 7-manifold. This way we recover eq. (48) in which the 4-dimensional Planck mass depends on the physical (and not coordinate) volume of the manifold on which we compactify. As a result, if we start from the product of the 4-dimensional Minkowski space and some 7-dimensional compact space (in the leading order of the expansion in  $\kappa^{2/3}$ ) as a ground state in  $d = 11$  we obtain the Minkowski space with the standard normalization as the vacuum in  $d = 4$ .

To find a more explicit formula for  $c_4$  we have to discuss the fields  $\sigma$  and  $\gamma$  in some detail. In the leading approximation  $\sigma$  is the overall modulus of the Calabi–Yau 6-manifold. We can divide it into a sum of the vacuum expectation value,  $\langle \sigma \rangle$ , and the fluctuation  $\tilde{\sigma}$ . In general both parts could depend on all 11 coordinates but in practice we have to impose some restrictions. The vacuum expectation value can not depend on  $x^\mu$  if the 4-dimensional theory is to be Lorentz-invariant. In the fluctuations we drop the dependence on the compact coordinates corresponding to the higher Kaluza–Klein modes. Furthermore, we know that in the leading approximation  $\langle \sigma \rangle$  is just a constant,  $\sigma_0$ , while corrections depending on the internal coordinates,  $\sigma_1$ , are of the next

order in  $\kappa^{2/3}$ . Thus, we obtain

$$\sigma(x^\mu, x^m, x^{11}) = \langle \sigma \rangle (x^m, x^{11}) + \tilde{\sigma}(x^\mu) = \sigma_0 + \sigma_1(x^m, x^{11}) + \tilde{\sigma}(x^\mu). \quad (55)$$

To make the above decomposition unique we define  $\sigma_0$  by requiring that the integral of  $\sigma_1$  over the internal space vanishes. The analogous decomposition can be also done for  $\gamma$ . With the above definitions the physical volume of the compact space is

$$V_7 = \int d^7x \langle e^{2\sigma} e^\gamma \rangle = e^{2\sigma_0} e^{\gamma_0} \hat{V}_7 \quad (56)$$

up to corrections of order  $\kappa^{4/3}$ . Thus, the parameter  $c_4$  can be written as

$$c_4 = e^{2\sigma_0} e^{\gamma_0}. \quad (57)$$

The choice of the coordinate volumes is just a convention. For example in the case of the Calabi–Yau 6–manifold only the product  $e^{3\sigma} \hat{V}_6$  has physical meaning. For definiteness we will use the convention that the coordinate volumes are equal 1 in  $M_{11}$  units. Thus,  $\langle e^{3\sigma} \rangle$  describes the Calabi–Yau volume in these units. Using eqs. (50,51) we obtain  $e^{3\sigma_0} = V M_{11}^6 \approx (2.3)^6$ ,  $e^{\gamma_0} e^{-\sigma_0} = R_{11} M_{11} \approx 9.2 a^2$ . The parameter  $c_4$  is equal to the square of the 4–dimensional Planck mass in these units and numerically  $c_4 \approx (35a)^2$ .

At the classical level we compactify on  $M^4 \times X^6 \times S^1/Z_2$ . This means that the vacuum expectation values  $\langle \sigma \rangle$  and  $\langle \gamma \rangle$  are just constants and eq. (55) reduces to

$$\sigma = \sigma_0 + \tilde{\sigma}(x^\mu), \quad \gamma = \gamma_0 + \tilde{\gamma}(x^\mu). \quad (58)$$

In such a situation  $\sigma$  and  $\gamma$  are 4–dimensional fields. We introduce two other 4–dimensional fields by the relations

$$\frac{1}{4!c_4} e^{6\sigma} G_{11\lambda\mu\nu} = \epsilon_{\lambda\mu\nu\rho} (\partial^\rho D), \quad (59)$$

$$C_{11a\bar{b}} = C_{11} \delta_{a\bar{b}} \quad (60)$$

where  $x^a$  ( $x^{\bar{b}}$ ) is the holomorphic (antiholomorphic) coordinate of the Calabi–Yau manifold. Now we can define the dilaton and the modulus fields by

$$\mathcal{S} = \frac{1}{(4\pi)^{2/3}} \left( e^{3\sigma} + i24\sqrt{2}D \right), \quad (61)$$

$$\mathcal{T} = \frac{1}{(4\pi)^{2/3}} \left( e^\gamma + i6\sqrt{2}C_{11} + C_i^* C_i \right) \quad (62)$$

where the observable sector matter fields  $C_i$  originate from the gauge fields  $A_M$  on the 10-dimensional observable wall (and  $M$  is an index in the compactified six dimensions). The Kähler potential takes its standard form

$$K = -\log(\mathcal{S} + \mathcal{S}^*) - 3\log(\mathcal{T} + \mathcal{T}^* - 2C_i^* C_i). \quad (63)$$

The imaginary part of  $\mathcal{S}$  ( $\text{Im}\mathcal{S}$ ) corresponds to the model independent axion, and with the above normalization the gauge kinetic function is  $f = \mathcal{S}$ . We have also

$$W(C) = d_{ijk} C_i C_j C_k \quad (64)$$

Thus the action to leading order is very similar to the weakly coupled case.

Before drawing any conclusion from the formulae obtained above we have to discuss a possible obstruction at the next to leading order. For the 3-index tensor field  $H$  in  $d = 10$  supergravity to be well defined one has to satisfy  $dH = \text{tr}F_1^2 + \text{tr}F_2^2 - \text{tr}R^2 = 0$  cohomologically. In the simplest case of the standard embedding one assumes  $\text{tr}F_1^2 = \text{tr}R^2$  locally and the gauge group is broken to  $E_6 \times E_8$ . Since in the M-theory case the two different gauge groups live on the two different boundaries (walls) of space-time such a cancellation point by point is no longer possible [115]. We expect nontrivial vacuum expectation values (vevs) of

$$(dG) \propto \sum_i \delta(x^{11} - x_i^{11}) \left( \text{tr}F_i^2 - \frac{1}{2}\text{tr}R^2 \right) \quad (65)$$

at least on one boundary ( $x_i^{11}$  is the position of  $i$ -th boundary). In the case of the standard embedding we would have  $\text{tr}F_1^2 - \frac{1}{2}\text{tr}R^2 = \frac{1}{2}\text{tr}R^2$  on one and  $\text{tr}F_2^2 - \frac{1}{2}\text{tr}R^2 = -\frac{1}{2}\text{tr}R^2$  on the other boundary. This might pose a severe problem since a nontrivial vev of  $G$  might be in conflict with supersymmetry ( $G_{11ABC} = H_{ABC}$ ). The supersymmetry transformation law in  $d = 11$  reads

$$\delta\psi_M = D_M\eta + \frac{\sqrt{2}}{288} G_{IJKL} \left( \Gamma_M^{IJKL} - 8\delta_M^I \Gamma^{JKL} \right) \eta + \dots \quad (66)$$

Supersymmetry will be broken unless e.g. the derivative term  $D_M\eta$  compensates the nontrivial vev of  $G$ . Witten has shown [115] that such a cancellation can occur and constructed the solution in the linearized approximation (linear in the expansion parameter  $\kappa^{2/3}$ ). This solution requires some modification of the metric on  $M^{11}$ :

$$g_{MN}^{(11)} = \begin{pmatrix} (1+b)\eta_{\mu\nu} & & \\ & (g_{ij} + h_{ij}) & \\ & & (1+\gamma') \end{pmatrix}. \quad (67)$$

$M^{11}$  is no longer a direct product  $M^4 \times X^6 \times S^1/Z_2$  because  $b$ ,  $h_{ij}$  and  $\gamma'$  depend now on the compactified coordinates. The volume of  $X^6$  depends on  $x^{11}$  [115]:

$$\frac{\partial}{\partial x^{11}} V = -\frac{\sqrt{2}}{8} \int d^6 x \sqrt{g} \omega^{AB} \omega^{CD} G_{ABCD} \quad (68)$$

where the integral is over the Calabi–Yau manifold  $X^6$  and  $\omega$  is the corresponding Kähler form. The parameter  $(1+b)$  is the scale factor of the Minkowski 4-manifold and depends on  $x^{11}$  in the following way

$$\frac{\partial}{\partial x^{11}} b = \frac{1}{2} \frac{\partial}{\partial x^{11}} \log v_4 = \frac{\sqrt{2}}{24} \omega^{AB} \omega^{CD} G_{ABCD} \quad (69)$$

where  $v_4$  is the physical volume for some fixed coordinate volume in  $M^4$ . In our simple reduction and truncation method with the metric  $g_{MN}^{(11)}$  given by eq. (52) we can reproduce the  $x^{11}$  dependence of  $V$  and  $v_4$ . The volume of  $X^6$  is determined by  $\sigma$ :

$$\frac{\partial}{\partial x^{11}} \log V = \frac{\partial}{\partial x^{11}} (3 \langle \sigma \rangle) = 3 \frac{\partial}{\partial x^{11}} \sigma \quad (70)$$

while the scale factor of  $M^4$  can be similarly expressed in terms of  $\sigma$  and  $\gamma$  fields:

$$\frac{\partial}{\partial x^{11}} \log v_4 = -\frac{\partial}{\partial x^{11}} (2 \langle \gamma \rangle + 4 \langle \sigma \rangle) = -\frac{\partial}{\partial x^{11}} (2\gamma + 4\sigma). \quad (71)$$

Substituting  $\langle \sigma \rangle$  with  $\sigma$  in the above two equations is allowed because, due to our decomposition (55), only the vev of  $\sigma$  depends on the internal coordinates (the same is true for  $\gamma$ ). The scale factor  $b$  calculated in ref. [115] depends also on the Calabi–Yau coordinates. Such a dependence can not be reproduced in our simple reduction and truncation compactification so we have to average eq. (69) over  $X^6$ . Using equations (68–71) after such an averaging we obtain (to leading order in the expansion parameter  $\kappa^{2/3}$ ) [101]

$$\frac{\partial \gamma}{\partial x^{11}} = -\frac{\partial \sigma}{\partial x^{11}} = \frac{\sqrt{2}}{24} \frac{\int d^6 x \sqrt{g} \omega^{AB} \omega^{CD} G_{ABCD}}{\int d^6 x \sqrt{g}}. \quad (72)$$

Substituting the vacuum expectation value of  $G$  found in [115] we can rewrite it in the form

$$\frac{\partial \gamma}{\partial x^{11}} = -\frac{\partial \sigma}{\partial x^{11}} = \frac{2}{3} \alpha \kappa^{2/3} V^{-2/3} \quad (73)$$

where

$$\alpha = \frac{\pi c}{2(4\pi)^{2/3}} \quad (74)$$

and  $c$  is a constant of order unity given for the standard embedding of the spin connection by

$$c = V^{-1/3} \left| \int \frac{\omega \wedge \text{tr}(R \wedge R)}{8\pi^2} \right|. \quad (75)$$

Our calculations, as those of Witten, are valid only in the leading non-trivial order in the  $\kappa^{2/3}$  expansion. The expression (73) for the derivatives of  $\sigma$  and  $\gamma$  contain an explicit factor  $\kappa^{2/3}$ . This means that we should take the lowest order value for the Calabi–Yau volume in that expression. An analogous procedure has been used in obtaining all formulae presented in this paper. We always expand in  $\kappa^{2/3}$  and drop all terms which are of higher order. Taking the above into account and using our units in which  $M_{11} = 1$  we can rewrite eq. (73) in the simple form:

$$\frac{\partial \gamma}{\partial x^{11}} = -\frac{\partial \sigma}{\partial x^{11}} = \frac{2}{3} \alpha e^{-2\sigma_0}. \quad (76)$$

Eqs. (72–76) as derived in ref. [101] contain all the information to deduce the effective action, i.e. Kähler potential, superpotential and gauge kinetic function of the 4-dimensional effective supergravity theory.

It is the above dependence of  $\sigma$  and  $\gamma$  on  $x^{11}$  that leads to these consequences. One has to be careful in defining the fields in  $d = 4$ . It is obvious, that the 4-dimensional fields  $\mathcal{S}$  and  $\mathcal{T}$  can not be any longer defined by eqs. (61, 62) because now  $\sigma$  and  $\gamma$  are 5-dimensional fields. We have to integrate out the dependence on the 11th coordinate. In the present approximation, this procedure is quite simple: we have to replace  $\sigma$  and  $\gamma$  in the definitions of  $\mathcal{S}$  and  $\mathcal{T}$  with their averages over the  $S^1/Z_2$  interval [101]. With the linear dependence of  $\sigma$  and  $\gamma$  on  $x^{11}$  their average values coincide with the values taken at the middle of the  $S^1/Z_2$  interval

$$\bar{\sigma} = \sigma \left( \frac{\pi \rho}{2} \right) = \sigma_0 + \tilde{\sigma}(x^\mu), \quad (77)$$

$$\bar{\gamma} = \gamma \left( \frac{\pi \rho}{2} \right) = \gamma_0 + \tilde{\gamma}(x^\mu). \quad (78)$$

When we reduce the boundary part of the Lagrangian of M-theory to 4 dimensions we find exponents of  $\sigma$  and  $\gamma$  fields evaluated at the boundaries. Using eqs. (55) and (76) we get

$$e^{-\gamma}|_{M_i^{10}} = e^{-\gamma_0} \pm \frac{1}{3} \alpha e^{-3\sigma_0}, \quad (79)$$



$$e^{3\sigma}|_{M_i^{10}} = e^{3\sigma_0} \pm \alpha e^{\gamma_0}. \quad (80)$$

The above formulae have very important consequences for the definitions of the Kähler potential and the gauge kinetic functions. For example, the coefficient in front of the  $D_\mu C_i^* D^\mu C_i$  kinetic term is proportional to  $e^{-\gamma}$  evaluated at the  $E_6$  wall where the matter fields propagate. At the lowest order this was just  $e^{-\gamma_0}$  or  $\langle \mathcal{T} \rangle^{-1}$  up to some numerical factor. From eq. (79) we see that at the next to leading order also  $\langle \mathcal{S} \rangle^{-1}$  is involved with relative coefficient  $\alpha/3$ . Taking such corrections into account we find that at this order the Kähler potential is given by

$$K = -\log(\mathcal{S} + \mathcal{S}^*) + \frac{2\alpha C_i^* C_i}{\mathcal{S} + \mathcal{S}^*} - 3\log(\mathcal{T} + \mathcal{T}^* - 2C_i^* C_i) \quad (81)$$

with  $\mathcal{S}$  and  $\mathcal{T}$  now defined by

$$\mathcal{S} = \frac{1}{(4\pi)^{2/3}} \left( e^{3\bar{\sigma}} + i24\sqrt{2}\bar{D} + \alpha C_i^* C_i \right), \quad (82)$$

$$\mathcal{T} = \frac{1}{(4\pi)^{2/3}} \left( e^{\bar{\gamma}} + i6\sqrt{2}\bar{C}_{11} + C_i^* C_i \right) \quad (83)$$

where bars denote averaging over the 11th dimension. It might be of some interest to note that the combination  $\mathcal{S}\mathcal{T}^3$  is independent of  $x^{11}$  even before this averaging procedure took place. The solution above is valid only for terms at most linear in  $\alpha$ . Keeping this in mind we could write the Kähler potential also in the form

$$K = -\log(\mathcal{S} + \mathcal{S}^* - 2\alpha C_i^* C_i) - 3\log(\mathcal{T} + \mathcal{T}^* - 2C_i^* C_i). \quad (84)$$

Equipped with this definition the calculation of the gauge kinetic function(s) from eqs. (76, 80) becomes a trivial exercise [101]. In the five-dimensional theory  $f$  depends on the 11-dimensional coordinate as well, thus the gauge kinetic function takes different values at the two walls. The averaging procedure allows us to deduce these functions directly. For the simple case at hand (the so-called standard embedding) eq. (80) gives [101]

$$f_6 = \mathcal{S} + \alpha\mathcal{T}; \quad f_8 = \mathcal{S} - \alpha\mathcal{T}. \quad (85)$$

It is a special property of the standard embedding that the coefficients are equal and opposite. The coefficients might vary for more general cases. This completes the discussion of the  $d = 4$  effective action in next to leading order, noting that the superpotential does not receive corrections at this level.

The nontrivial dependence of  $\sigma$  and  $\gamma$  on  $x^{11}$  can also enter definitions and/or interactions of other 4-dimensional fields. Let us next consider

the gravitino. After all we have to show that this field is massless to give the final proof that the given solution respects supersymmetry. Its 11-dimensional kinetic term

$$-\frac{1}{2}\sqrt{g}\bar{\psi}_I\Gamma^{IJK}D_J\psi_K \quad (86)$$

remains diagonal after compactification to  $d = 4$  if we define the 4-dimensional gravitino,  $\psi_\mu^{(4)}$ , and dilatino,  $\psi_{11}^{(4)}$ , fields by the relations

$$\psi_\mu = e^{-(\sigma-\sigma_0)/2}e^{-(\gamma-\gamma_0)/4}\left(\psi_\mu^{(4)} + \frac{1}{\sqrt{6}}\Gamma_\mu\psi_{11}^{(4)}\right), \quad (87)$$

$$\psi_{11} = -\frac{2}{\sqrt{6}}e^{(\sigma-\sigma_0)/2}e^{(\gamma-\gamma_0)/4}\Gamma^{11}\psi_{11}^{(4)}. \quad (88)$$

The  $d = 11$  kinetic term (86) gives after the compactification also a mass term for the  $d = 4$  gravitino of the form

$$\frac{3}{8}e^{\sigma_0}e^{-\gamma_0}\frac{\partial\gamma}{\partial x^{11}} = \frac{\sqrt{2}}{64}e^{\sigma_0}e^{-\gamma_0}\frac{\int d^6x\sqrt{g}\omega^{AB}\omega^{CD}G_{ABCD}}{\int d^6x\sqrt{g}} = \frac{1}{4}\alpha e^{-\sigma_0}e^{-\gamma_0}. \quad (89)$$

The sources of such a term are nonzero values of the spin connection components  $\omega_\mu^{\alpha 11}$  and  $\omega_m^{a 11}$  resulting from the  $x^{11}$  dependence of the metric. It is a constant mass term from the 4-dimensional point of view. This, however, does not mean that the gravitino mass is nonzero. There is another contribution from the 11-dimensional term

$$-\frac{\sqrt{2}}{384}\sqrt{g}\bar{\psi}_I\Gamma^{IJKLMN}\psi_N\left(G_{JKLM} + \hat{G}_{JKLM}\right). \quad (90)$$

After redefining fields according to (87,88) and averaging the nontrivial vacuum expectation value of  $G$  over  $X^6$  we get from eq. (90) a mass term which exactly cancels the previous contribution (89). The gravitino is massless – the result which we expect in a model with unbroken supersymmetry and vanishing cosmological constant. Thus, we find that our simple reduction and truncation method (including the correct  $x^{11}$  dependence in next to leading order) reproduces the main features of the model.

The factor  $\langle \exp(3\sigma) \rangle$  represents the volume of the six-dimensional compact space in units of  $M_{11}^{-6}$ . The  $x^{11}$  dependence of  $\sigma$  then leads to the geometrical picture that the volume of this space varies with  $x^{11}$  and differs at the two boundaries:

$$V_{E_8} = V_{E_6} - 2\pi^2\rho\left(\frac{\kappa}{4\pi}\right)^{2/3}\left|\int\omega\wedge\frac{\text{tr}(F\wedge F) - \frac{1}{2}\text{tr}(R\wedge R)}{8\pi^2}\right| \quad (91)$$

where the integral is over  $X^6$  at the  $E_6$  boundary. In the given approximation, this variation is linear, and for growing  $\rho$  the volume on the  $E_8$  side becomes smaller and smaller. At a critical value of  $\rho$  the volume will thus vanish and this will provide us with an upper limit on  $\rho$ :

$$\rho < \rho_{crit} = \frac{(4\pi)^{2/3}}{c\pi^2} M_{11}^3 V_{E_6}^{2/3} \quad (92)$$

where  $c$  was defined in eq. (75). The critical value is model dependent and we shall not discuss this in detail here.

Let us now compare the M-theory picture with that of the weakly coupled heterotic string. Inspection of (45) and (85) reveals a close connection between the two [7, 104]. The variation of the Calabi–Yau manifold volume as discussed above is the analogue of the one loop correction of the gauge kinetic function (45) in the weakly coupled case and has the same origin, namely a Green–Schwarz anomaly cancellation counterterm. In fact, also in the strongly coupled case this leads to a correction for the gauge coupling constants at the  $E_6$  and  $E_8$  side. As seen, gauge couplings are no longer given by the (averaged)  $\mathcal{S}$ -field, but by that combination of the (averaged)  $\mathcal{S}$  and  $\mathcal{T}$  fields which corresponds to the  $\mathcal{S}$ -field before averaging at the given boundary leading to

$$f_{6,8} = \mathcal{S} \pm \alpha\mathcal{T} \quad (93)$$

at the  $E_6$  ( $E_8$ ) side respectively. The critical value of  $R_{11}$  will correspond to infinitely strong coupling at the  $E_8$  side  $\mathcal{S} - \alpha\mathcal{T} = 0$ . Since we are here close to criticality a correct phenomenological fit of  $\alpha_{\text{GUT}} = 1/25$  should include this correction  $\alpha_{\text{GUT}}^{-1} = \mathcal{S} + \alpha\mathcal{T}$  where  $\mathcal{S}$  and  $\alpha\mathcal{T}$  give comparable contributions. This is a difference to the weakly coupled case, where in  $f = S + \epsilon T$  the latter contribution was small compared to  $S$ . The stability of this result for the corrections to  $f$  when going from weak coupling to strong coupling is only possible because of the rather special properties of  $f$ .  $f$  does not receive further perturbative corrections beyond one loop [107, 95], and the one loop corrections are determined by the anomaly considerations. The formal expressions for the corrections are identical, the difference being only that in the strongly coupled case these corrections are to be interpreted of comparable importance as the classical value.

## 8. Supersymmetry breaking at the hidden wall

For the discussion of supersymmetry breakdown we should carefully examine the supersymmetry transformation of fermionic fields. Of particular importance are the fields that originate from the higher dimen-

sional gravitino. For the  $d = 11$  action, the supersymmetry transformation laws for these fields are given by

$$\begin{aligned} \delta\psi_A &= D_A\eta + \frac{\sqrt{2}}{288}G_{IJKL}\left(\Gamma_A^{IJKL} - 8\delta_A^I\Gamma^{JKL}\right)\eta - \\ &- \frac{1}{1152\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta(x^{11})(\bar{\chi}^a\Gamma_{BCD}\chi^a)\left(\Gamma_A^{BCD} - 6\delta_A^B\Gamma^{CD}\right)\eta\ldots \end{aligned} \quad (94)$$

as well as

$$\begin{aligned} \delta\psi_{11} &= D_{11}\eta + \frac{\sqrt{2}}{288}G_{IJKL}\left(\Gamma_{11}^{IJKL} - 8\delta_{11}^I\Gamma^{JKL}\right)\eta + \\ &+ \frac{1}{1152\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta(x^{11})(\bar{\chi}^a\Gamma_{ABC}\chi^a)\Gamma^{ABC}\eta + \ldots \end{aligned} \quad (95)$$

where gaugino bilinears appear in the right hand side of both expressions. Again we consider gaugino condensation at the hidden  $E_8$  boundary

$$\langle\bar{\chi}^a\Gamma_{ijk}\chi^a\rangle = g_8^2\Lambda^3\epsilon_{ijk}. \quad (96)$$

The  $E_8$  gauge coupling constant appears in this equation because the straightforward reduction and truncation leaves a non-canonical normalization for the gaugino kinetic term. An important property of the weakly coupled case (d=10 Lagrangian) was the fact that the gaugino condensate and the three-index tensor field  $H$  contributed to the scalar potential in a full square. Hořava made the important observation that a similar structure appears in the M-theory Lagrangian as well [52]:

$$-\frac{1}{12\kappa^2}\int_{M^{11}}d^{11}x\sqrt{g}\left(G_{ABC11} - \frac{\sqrt{2}}{32\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\delta(x^{11})\bar{\chi}^a\Gamma_{ABC}\chi^a\right)^2 \quad (97)$$

with the obvious relation between  $H$  and  $G$ . Let us now have a closer look at the form of  $G$ . At the next to leading order we have

$$\begin{aligned} G_{11ABC} &= (\partial_{11}C_{ABC} + \text{permutations}) \\ &+ \frac{1}{4\pi\sqrt{2}}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_i\delta(x^{11}-x_i^{11})(\omega_{ABC}^{YM} - \frac{1}{2}\omega_{ABC}^L). \end{aligned} \quad (98)$$

Observe, that in the bulk we have  $G = dC$  with the Chern–Simons contributions confined to the boundaries. Formula (97) suggests a cancellation between the gaugino condensate and the  $G$ -field in a way very similar to the weakly coupled case, but the nature of the cancellation of the terms becomes much more transparent now. Remember that in the former case we had argued that because of the quantization condition

for  $\langle dB \rangle$  the gaugino condensate is cancelled not by  $\langle dB \rangle$  but by a flux of the Chern–Simons terms. *Here this becomes obvious.* The condensate is located at the wall as are the Chern–Simons terms, so this cancellation has to happen locally at the wall and  $dC$  should vanish for  $G$  not to have a vev in the bulk. In any case there is a quantization condition for  $dC$  as well [116].

So this cancellation is very similar to the one in the weakly coupled case. At the minimum of the potential we obtain  $G_{ABCD} = 0$  everywhere and

$$G_{ABC11} = \frac{\sqrt{2}}{32\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) \bar{\chi}^a \Gamma_{ABC} \chi^a \quad (99)$$

at the hidden wall. Eqs. (94) and (95) then become

$$\delta\psi_A = D_A \eta + \dots \quad (100)$$

$$\delta\psi_{11} = D_{11} \eta + \frac{1}{384\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) (\bar{\chi}^a \Gamma_{ABC} \chi^a) \Gamma^{ABC} \eta \dots \quad (101)$$

An inspection of the potential shows that  $\delta\psi_{11}$  is nonvanishing and supersymmetry is spontaneously broken. Because of the cancellation in eq. (97), the cosmological constant vanishes to leading order. Recalling supersymmetry transformation law for the elfbein

$$\delta e_I^m = \frac{1}{2} \bar{\eta} \Gamma^m \psi_I, \quad (102)$$

one finds that the superpartner of the  $\mathcal{T}$  field plays the role of the goldstino. Again we have a situation where  $F_{\mathcal{S}} = 0$  (due to the cancellation in (97)) with nonvanishing  $F_{\mathcal{T}}$ . But here we find the novel and interesting situation that  $F_{\mathcal{T}}$  differs from zero only at the hidden wall, although the field itself is a bulk field.

At that wall our discussion is completely 4-dimensional although we are still dealing effectively with a  $d = 5$  theory. To reach the effective theory in  $d = 4$  we have to integrate out the dependence of the  $x^{11}$  coordinate. As in the previous section this can be performed by the averaging procedure explained there. With the gaugino condensation scale  $\Lambda$  sufficiently small compared to the compactification scale  $M_{GUT}$ , the low-energy effective theory is well described by four dimensional  $N = 1$  supergravity in which supersymmetry is spontaneously broken. In this case, the modes which remain at low energies will be well approximated by constant modes along the  $x^{11}$  direction. This observation justifies our averaging procedure to obtain four dimensional quantities. Averaging  $\delta\psi_{11}$  over  $x^{11}$ , we thus obtain the vev of the auxiliary field  $F_{\mathcal{T}}$

$$F_{\mathcal{T}} = \frac{1}{2} \mathcal{T} \frac{\int dx^{11} \sqrt{g_{1111}} \delta\psi_{11}}{\int dx^{11} \sqrt{g_{1111}}}. \quad (103)$$

Note that this procedure allows for a nonlocal cancellation of the vev of the auxiliary field in  $d = 4$ . A condensate with equal size and opposite sign at the observable wall could cancel the effect and restore supersymmetry. Using  $\int dx^{11} \sqrt{g_{1111}} \delta(x^{11}) = 1$ , the auxiliary field is found to be

$$F_{\mathcal{T}} = \mathcal{T} \frac{1}{32\pi(4\pi)^{2/3}} \frac{g_8^2 \Lambda^3}{R_{11} M_{11}^3} \quad (104)$$

Similarly one can easily show that  $F_{\mathcal{S}}$  as well as the vacuum energy vanish. This allows us then to unambiguously determine the gravitino mass, which is related to the auxiliary field in the following way:

$$m_{3/2} = \frac{F_{\mathcal{T}}}{\mathcal{T} + \mathcal{T}^*} = \frac{1}{64\pi(4\pi)^{2/3}} \frac{g_8^2 \Lambda^3}{R_{11} M_{11}^3} = \frac{\pi}{2} \frac{\Lambda^3}{M_{Planck}^2}. \quad (105)$$

As a nontrivial check one may calculate the gravitino mass in a different way. A term in the Lagrangian

$$-\frac{\sqrt{2}}{192\kappa^2} \int dx^{11} \sqrt{g} \bar{\psi}_I \Gamma^{IJKLMN} \psi_N G_{JKLM}, \quad (106)$$

becomes the gravitino mass term when compactified to four dimensions. Using the vevs of the  $G_{IJK11}$  given by eq. (99), one can obtain the same result as eq. (105). This is a consistency check of our approach and the fact that the vacuum energy vanishes in the given approximation.

It follows from eq. (105), that the gravitino mass tends to zero when the radius of the eleventh dimension goes to infinity. When the four-dimensional Planck scale is fixed to be the measured value, however, the gravitino mass in the strongly coupled case is expressed in a standard manner, similar to the weakly coupled case. To obtain the gravitino mass of the order of 1 TeV, one has to adjust  $\Lambda$  to be of the order of  $10^{13}$  GeV when one constructs a realistic model by appropriately breaking the  $E_8$  gauge group at the hidden wall.

In the minimization of the potential we have implicitly used the leading order approximation. As was explained in a previous section, the next to leading order correction gives the non-trivial dependence of the background metric on  $x^{11}$ . Then the Einstein–Hilbert action in eleven dimensions gives additional contribution to the scalar potential in the four-dimensional effective theory, which shifts the vevs of the  $G_{IJKL}$ . As a consequence,  $F_{\mathcal{S}}$  will no longer vanish. Though this may be significant when we discuss soft masses, it does not drastically change our estimate of the gravitino mass (105) and our main conclusion drawn here is still valid after the higher order corrections are taken into account. In any case, these questions have to be addressed if one aims at realistic models for particle physics.

## 9. Summary and outlook

In these lectures we have discussed the mechanism of gaugino condensation and flux stabilization within the heterotic scenario in its simplest version. The picture could be easily generalized to type II orientifolds or other types of string constructions, leading to the notion of supersymmetry breakdown on a hidden brane.

There is, however, still a long way to go towards realistic model building. One of the obstacles is the question of moduli stabilization in string theory. Without a solution to this problem we shall usually obtain so-called runaway vacua where e.g. coupling constants run to unrealistic values. Another obstacle is the appearance of instabilities of the scalar potential once we include radiative corrections. We have discussed some aspects of this in section 6 when we included radiative (threshold) corrections to the gauge coupling constants (45). If we would consider  $f = S \pm \epsilon T$  and reinsert this into the  $F$ -terms in (32) we would obtain new contributions to the scalar potential leading to minima with a negative vacuum energy. This, of course, is nothing else than the problem of the cosmological constant.

Recently there has been some revived interest in this discussion. One aspect concerns the consideration of compactification of the extra dimension on non-Kähler manifolds. Moduli are stabilized with the help of fluxes of various antisymmetric tensor fields. For more details and references see [9, 15, 22, 44, 63]. This allows the stabilization of many moduli already in the supersymmetric framework in a rather general context and avoids cosmological moduli problems.

Within the heterotic M-theory context there have been attempts to go beyond the classical level [23]. In ref. [13] one finds a rather comprehensive discussion of moduli stabilization and supersymmetry breakdown in a general set-up of heterotic M-theory including 5-branes in the bulk. Moduli can be stabilized, but a large negative vacuum energy remains. Similar results in the heterotic string theory have been reported in [51]. These results are, of course, also of interest in the discussion of cosmological aspects of string theory [62]. So the mechanism of supersymmetry breakdown through gaugino condensation still remains one of the most promising subjects in the discussion of realistic string model building.

## Acknowledgements

We would like to thank the organizers, especially Gerard Smadja, for their hospitality. This work was partially supported by the European Community's Human Potential Programme under contracts HPRN-CT-2000-00131 Quantum Space-time, HPRN-CT-2000-00148 Physics Across

the Present Energy Frontier and HPRN-CT-2000-00152 Supersymmetry and the Early Universe.

## References

- [1] Aldazabal, G., Font, A., Ibáñez, L. E. and Uranga, A.M., *String GUTs*, *Nucl. Phys. B* **452** (1995) 3
- [2] Amaldi, U., de Boer, W., Fürstenau, H., *Phys. Lett. B* **281** (1992) 374; Antoniadis, I., Ellis, J., Kelley, S., and Nanopoulos, D. V., *Phys. Lett. B* **272** (1991) 31
- [3] Antoniadis, I., Ellis, J., Lacaze, R. and Nanopoulos, D. V., *Phys. Lett. B* **268** (1991) 188; Dolan, L. and Liu, J. T., *Nucl. Phys. B* **387** (1992) 86
- [4] Antoniadis, I., Gava, E., Narain K. S. and Taylor, T. R., *Nucl. Phys. B* **432** (1994) 187
- [5] Antoniadis, I. , Narain, K. S. and Taylor, T. R., *Phys. Lett. B* **267** (1991) 37; Antoniadis, I., Gava, E. and Narain, K. S., *Nucl. Phys. B* **383** (1992) 93; *Phys. Lett. B* **283** (1992) 209; Mayr, P., and Stieberger, S., *Nucl. Phys. B* **412** (1994) 502
- [6] Antoniadis, I. and Quirós, M., *Phys. Lett. B* **392** (1997) 61; hep-th/9705037
- [7] Banks, T. and Dine, M., *Nucl. Phys. B* **479** (1996) 173
- [8] Banks, T. and Dine, M., *Nucl. Phys. B* **505** (1997) 445
- [9] K. Becker, M. Becker, K. Dasgupta and P. S. Green, *JHEP* **0304**, 007 (2003) [arXiv:hep-th/0301161].
- [10] Bershadsky, M., Cecotti, Ooguri, S. H. and Vafa, C., *Nucl. Phys. B* **405** (1993) 279; *Comm. Math. Phys.* **165** (1994) 311; Hosono, S., Klemm, A., Theisen, S. and Yau, S. T., *Nucl. Phys. B* **433** (1995) 501
- [11] Binetruy, P. and Gaillard, M.K., *Phys. Lett. B* **232** (1989) 83
- [12] Brignole, A., Ibáñez, L. E. and Muñoz, C., *Nucl. Phys. B* **422** (1994) 125 and references therein
- [13] E. I. Buchbinder and B. A. Ovrut, arXiv:hep-th/0310112.
- [14] Candelas, P., Horowitz, G., Strominger A. and Witten, E., *Nucl. Phys. B* **258** (1985) 46
- [15] G. L. Cardoso, G. Curio, G. Dall’Agata and D. Lust, arXiv:hep-th/0310021.
- [16] Chamseddine, A. H., *Phys. Rev. D* **24** (1981) 3065; Chapline, G. F. and Manton, N. S., *Phys. Lett. B* **120** (1983) 105
- [17] Choi, K., *Phys. Rev. D* **56** (1997) 6588
- [18] Choi, K. and Kim. J. E., *Phys. Lett. B* **165** (1985) 71
- [19] Choi, K., Kim, H. B. and Muñoz, C., *Phys. Rev. D* **57** (1998) 7521, hep-th/9711158
- [20] Conrad, J. O., *Phys. Lett. B* **421** (1998) 119, hep-th/9708031
- [21] Cremmer, E., Ferrara, S., Girardello, L. and van Proeyen, A., *Nucl. Phys. B* **212** (1983) 413



- [22] G. Curio and A. Krause, Nucl. Phys. B **643**, 131 (2002) [arXiv:hep-th/0108220].
- [23] G. Curio and A. Krause, arXiv:hep-th/0308202.
- [24] Cvetič, M., Font, A., Ibáñez, L. E., Lüster, D. and Quevedo, F., *Nucl. Phys. B* **361** (1991) 194
- [25] For an extended list of references see: Carlos, B. de, Casas, J. A., Muñoz, C., Nucl.Phys. **B399** (1993) 623
- [26] Font, A., Ibáñez, L. E., Lüster, D., Quevedo, F., Phys.Lett. **249B** (1990) 35
- [27] Derendinger, J. P., Ferrara, S., Kounnas, C. and Zwirner, F., *Nucl. Phys. B* **372** (1992) 145;  
Antoniadis, I., Gava, E., Narain, K. S. and Taylor, T. R., *Nucl. Phys. B* **407** (1993) 706
- [28] Derendinger, J. P., Ibáñez, L. E. and Nilles, H. P., Phys. Lett. **B155** (1985) 65
- [29] Derendinger, J. P., Ibáñez, L. E. and Nilles, H. P., Nucl. Phys. **B267** (1986) 365
- [30] Dienes, K. R. and Faraggi, A. E., *Making ends meet: string unification and low-energy data*, Princeton IASSNS-HEP-95/24 (hep-th/9505018); *Gauge coupling unification in realistic free-fermionic string models*, Princeton IASSNS-HEP-94/113 (hep-th/9505046)
- [31] Dine, M., Fischler, W., Srednicki, M., Nucl.Phys. **B189** (1981) 575
- [32] Dine, M., Rohm, R., Seiberg, N. and Witten, E., Phys. Lett. **B156** (1985) 55
- [33] Dine, M., Seiberg, N., Wen, X. G. and Witten, E., Nucl. Phys. **B289** (1987) 319;  
Nucl. Phys. **B278** (1986) 769
- [34] Dixon, L., Harvey, J., Vafa, C. and Witten, E., *Nucl. Phys. B* **261** (1985) 678;  
**B 274** (1986) 285;  
Ibáñez, L. E., Mas, J., Nilles, H. P. and Quevedo, F., *Nucl. Phys. B* **301** (1988) 157
- [35] Dixon, L., Kaplunovsky, V. and Louis, J. *Nucl. Phys. B* **355** (1991) 649
- [36] Dudas, E., *Phys. Lett. B* **B416** (1998) 309, hep-th/9709043
- [37] Dudas, E. and Grojean, C., *Nucl. Phys. B* **B507** (1997) 553, hep-th/9704177
- [38] Ellis, J., Kim, J. E. and Nanopoulos, D. V., Phys. Lett. **B145** (1984) 181
- [39] Ellis, J., Kelley, S. and Nanopoulos, D. V., *Phys. Lett. B* **249** (1990) 441;  
Amaldi, U., Boer, W. de and Fürstenau, H., *Phys. Lett. B* **260** (1991) 447;  
Langacker, P. and Luo, M. X., *Phys. Rev. D* **44** (1991) 817
- [40] Ferrara, S., Girardello, L. and Nilles, H. P., Phys. Lett. **B125** (1983) 457
- [41] Ferrara, S., Kounnas, C., Lüster, D. and Zwirner, F., *Nucl. Phys. B* **365** (1991) 431
- [42] Font, A., Ibáñez, L. E., D. Lüster, Quevedo, F., Phys.Lett. **245B** (1990) 401;  
Ferrara, S., Magnoli, N., Taylor, T. R., Veneziano, G., Phys.Lett. **245B** (1990) 409;  
Nilles, H. P., M. Olechowski, Phys.Lett. **248B** (1990) 268;  
Binetruy, P., Gaillard, M. K., Phys.Lett. **253B** (1991) 119;  
Cvetič, M., Font, A., Ibáñez, L. E., Lüster, D., Quevedo, F., Nucl.Phys. **B361** (1991) 194
- [43] Georgi, H., Kim, Jihn E., Nilles, H. P., *Phys. Lett. B* **B437** (1998) 325, hep-ph/9805510

- [44] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097].
- [45] Ginsparg, P. *Phys. Lett. B* **197** (1987) 139
- [46] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, Nucl. Phys. B **256**, 253 (1985).
- [47] Horne, J. H., Moore, G., Nucl.Phys. **B432** (1994) 109
- [48] Ibáñez, L. E., Nilles, H. P., Phys.Lett. **169B** (1986) 354;  
Dixon, L., Kaplunovsky, V., Louis, J., Nucl.Phys. **B355** (1991) 649
- [49] Green, M. and Schwarz, J., Phys. Lett. **B149** (1984) 117
- [50] Green, M., Schwarz, J. and Witten, E., Superstring Theory, Cambridge University Press, 1987
- [51] S. Gukov, S. Kachru, X. Liu and L. McAllister, arXiv:hep-th/0310159.
- [52] Hořava, P., Phys. Rev. **D54** (1996) 7561
- [53] Hořava, P. and Witten, E., Nucl. Phys. **B460** (1996) 506; Nucl. Phys. **B475** (1996) 94.
- [54] Ibáñez, L. E., *Phys. Lett. B* **318** (1993) 73
- [55] Ibáñez, L. E. and Lüst, D., *Nucl. Phys. B* **382** (1992) 305
- [56] Ibáñez, L. E., Lüst, D. and Ross, G. G., *Phys. Lett. B* **272** (1991) 251
- [57] Ibáñez, L. E. and Nilles, H. P., Phys. Lett. **B169** (1986) 354
- [58] Ibáñez, L. E., Nilles, H. P. and Quevedo, F., *Phys. Lett. B* **187** (1987) 25;  
Ibáñez, L. E., Kim, J. E., Nilles, H. P. and Quevedo, F., *Phys. Lett. B* **191** (1987) 283
- [59] Ibáñez, L. E., Nilles, H. P. and Quevedo, F., *Phys. Lett. B* **192** (1987) 332
- [60] Jungman, G., Kamionkowski, M. and Griest, K., Phys. Rep. **267** (1996) 195
- [61] Kaplunovsky, V. S., *Nucl. Phys. B* **307** (1988) 145, Erratum: *Nucl. Phys. B* **382** (1992) 436
- [62] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].
- [63] S. Kachru, M. B. Schulz and S. Trivedi, JHEP **0310**, 007 (2003) [arXiv:hep-th/0201028].
- [64] Kaplunovsky, V. S. and Louis, J., Phys. Lett. **B306** (1993) 269
- [65] Kaplunovsky, V. S. and Louis, J., *Nucl. Phys. B* **444** (1995) 191
- [66] Kawamura, Y., Nilles, H. P., Olechowski, M. and Yamaguchi, M., **JHEP** **9806:008** (1998), hep-ph/9805397,
- [67] Kawasaki, M. and Moroi, T., Prog. Theor. Phys. **93** (1995) 879
- [68] Kawasaki, M. and Yanagida, T., Prog. Theor. Phys. **97** (1997) 809
- [69] Kiritsis, E. and Kounnas, C., *Nucl. Phys. B* **41** [Proceedings Sup.] (1995) 331; *Nucl. Phys. B* **442** (1995) 472; *Infrared-regulated string theory and loop corrections to coupling constants*, hep-th/9507051
- [70] Kim, J. E. and Nilles, H. P., Phys. Lett. B **553** (2003) 1 [arXiv:hep-ph/0210402].
- [71] Krasnikov, N. V., Phys.Lett. **193B** (1987) 37;  
Casas, J. A., Lalak, Z., Muñoz, C., Ross, G. G., Nucl.Phys. **B347** (1990) 243

- [72] Lalak, Z., Niemeyer, A., Nilles, H. P., Phys.Lett. **349B** (1995) 99
- [73] Lalak, Z., Niemeyer, A., Nilles, H. P., hep-th/9503170, Nucl.Phys. **B453** (1995) 100
- [74] Lalak, Z. and Thomas, S., Nucl. Phys. B **515** (1998) 55 hep-th/9707223
- [75] For a review see: Langacker, P., *Grand Unification and the Standard Model*, hep-ph/9411247
- [76] Langacker, P. and Polonsky, N., *Phys. Rev. D* **47** (1993) 4028 and references therein
- [77] Lauer, J., Mas, J. and Nilles, H. P., Phys. Lett. B **226** (1989) 251.
- [78] Lauer, J., Mas, J. and Nilles, H. P., Nucl. Phys. **B351** (1991) 353
- [79] Li, T., Lopez, J. L. and Nanopoulos, D. V., Mod. Phys. Lett. A **12** (1997) 2647, hep-ph/9702237
- [80] Lopes Cardoso, G., Lüst, D. and Mohaupt, T., *Nucl. Phys. B* **450** (1995) 115
- [81] Lukas, A., Ovrut, B. A. and Waldram, D., Nucl. Phys. B **532** (1998) 43, hep-th/9710208
- [82] Lukas, A., Ovrut, B. A. and Waldram, D., Phys. Rev. D **57** (1998) 7529, hep-th/9711197
- [83] Macorra, A. de la, Ross, G.G., Nucl.Phys. **B404** (1993) 321
- [84] Matalliatakis, D., Nilles, H. P., Theisen, S., hep-th/9710247; *Phys. Lett. B* **421** (1998) 169
- [85] Mayr, P., Nilles, H. P. and Stieberger, S., *Phys. Lett. B* **317** (1993) 53
- [86] Mayr, P. and Stieberger, S., *Nucl. Phys. B* **407** (1993) 725;  
Bailin, D. , Love, A., Sabra, W. A. and Thomas, S., *Mod. Phys. Lett. A* **9** (1994) 67; **A10** (1995) 337
- [87] Mayr, P. and Stieberger, S., *Phys. Lett. B* **355** (1995) 107
- [88] Mayr, P. and Stieberger, S., TUM-HEP-212/95;  
Stieberger, S., *One-loop corrections and gauge coupling unification in superstring theory*, Ph.D. thesis, TUM-HEP-220/95
- [89] For a review see: Mayr, P. and Stieberger, S., Proceedings *28th International Symposium on Particle Theory*, p. 72–79, Wendisch–Rietz (1994) (hep-th/9412196, DESY 95–027
- [90] Montonen, C., Olive, D., Phys.Lett. **72B** (1977) 117;  
Seiberg, N., Witten, E., Nucl.Phys. **B426** (1994) 19
- [91] Nilles, H. P., Is Supersymmetry Afraid Of Condensates?, Phys.Lett. **112B** (1982) 455
- [92] Nilles, H. P., Dynamically Broken Supergravity And The Hierarchy Problem, Phys. Lett. **B115** (1982) 193
- [93] Nilles, H. P., Supergravity Generates Hierarchies, Nucl.Phys. **B217** (1983) 366
- [94] Nilles, H. P., Supersymmetry, Supergravity And Particle Physics, Physics Reports **110** (1984) 1
- [95] Nilles, H. P., The Role Of Classical Symmetries In The Low-Energy Limit Of Superstring Theories, Phys. Lett. **B180** (1986) 240

- [96] Nilles, H. P., Lectures at the Trieste Spring School on Supersymmetry, Supergravity and Superstrings 1986, Eds. B. de Wit et al., World Scientific (1986), page 37
- [97] H. P. Nilles, Vacuum Degeneracy In Four-Dimensional String Theories, Nucl. Phys. Proc. Suppl. **5B**, 185 (1988).
- [98] Nilles, H. P., Int. Journ. of Modern Physics **A5** (1990) 4199
- [99] Nilles, H. P., TASI lectures 1990, Testing the Standard Model, Ed. M. Cvetič and P. Langacker, World Scientific 1991, page 633
- [100] Nilles, H. P., TASI lectures 1993, The Building Blocks of Creation, Ed. S. Raby and T. Walker, World Scientific 1994, page 291
- [101] Nilles, H. P., Olechowski. M. and Yamaguchi, M., hep-th/9707143, Phys. Lett. **B415** (1997) 24.
- [102] Nilles, H. P., Olechowski. M. and Yamaguchi, M., Nucl. Phys. B **530** (1998) 43, hep-th/9801030
- [103] Nilles, H. P. and Stieberger, S., *How to reach the correct  $\sin^2 \theta_W$  and  $\alpha_s$  in string theory*, hep-th/9510009, Phys.Lett. **B367** (1996) 126
- [104] Nilles, H. P. and Stieberger, S., hep-th/9702110, Nucl. Phys. **B499** (1997) 3
- [105] Pagels, H. and Primack, J. R., Phys. Rev. Lett **48** (1982) 223
- [106] Rohm, R. and Witten, E., Ann. Physics **170** (1986) 454
- [107] Shifman, M. and Vainshtein, A., Nucl. Phys. **B277** (1986) 456
- [108] Taylor, T. R., Phys.Lett. **164B** (1985) 43
- [109] G. 't Hooft, '*Naturalness, chiral symmetry and spontaneous chiral symmetry breaking*', in 'Recent Developments in Gauge Theories', Cargèse 1979, G. 't Hooft et al, New York 1980, *Plenum Press*
- [110] Veneziano, G., Yankielowicz, S., Phys.Lett. **113B** (1982) 231
- [111] Weinberg, S., Phys. Rev. Lett. **48** (1982) 1303
- [112] Weinberg, S., *Phys. Lett.* **B 91** (1980) 51
- [113] Witten, E., Constraints On Supersymmetry Breaking, Nucl.Phys. **B202** (1982) 253
- [114] Witten, E., Dimensional Reduction Of Superstring Models, Phys. Lett. **B155** (1985) 151
- [115] Witten, E., Strong Coupling Expansion Of Calabi-Yau Compactification, Nucl. Phys. **B471** (1996) 135
- [116] Witten, E., hep-th/9609122, J. Geom. Phys. **22** (1997) 1
- [117] Witten, E., Nucl.Phys. **B188** (1981) 513
- [118] Witten, E., Symmetry Breaking Patterns In Superstring Models, *Nucl. Phys.* **B 258** (1985) 75
- [119] Witten, E., New Issues In Manifolds Of SU(3) Holonomy, *Nucl. Phys.* **B 268** (1986) 79